

Homework for Econ 312 Chapter 5

May 5, 2026

1. Consider the testing of (rather simple) random walk theory of asset prices. This is motivated by a classic paper of Holbert Working. Suppose the price of an asset X_t takes a random walk $X_{t+1} = X_t + BW_{t+1}$, where W_{t+1} is a standardized univariate normal distribution and successive W_{t+j} 's are i.i.d. A researcher wants to test the random walk hypothesis. A database reports not X_t but a two-period moving average $Z_t = .5(X_t + X_{t-1})$.
 - (a) Represent the state and measurement equation in the form used in class and the online chapter to which we can apply the Kalman filter.
 - (b) Find the recursive solution to the Kalman filter for this example.
 - (c) Will the Kalman gain converge as the number of observations increases? If so, find the limiting formula and limiting recursive representation of the solutions.
2. Chapter 5.2 shows how to compute the recursive updating formula for constructing the distribution of the date $t + 1$ hidden state given data observed up to time $t + 1$. This computation depends on coefficient vectors and matrices. Suppose that these have to be estimated but can be parameterized in terms of some underlying vector θ . Use the recursive solution to represent the density for Z_{t+1} conditioned on date t information. Then show how to construct the date $t + 1$ contribution to the log-likelihood function conditioned on current and past Z 's.