Homework for Econ 312 Chapter 10

May 8, 2026

1. (Robust prediction under model misspecification) A decision rule is chosen to forecast

$$f(w, z) = f_1(z) + f_2(z)w$$

where the probability distribution over the w's is a standard normal. Consider decision rules that express δ as a function of the data z. A prize rule gives the implied forecast error, $\gamma(x) = f(w, z) - \delta(z)$. Take the utility function to be:

$$-\frac{1}{2}\gamma(x)^2 = -\frac{1}{2}[f(w, z) - \delta(z)]^2.$$

Assume that $\xi_2 > [f_2(z)]^2$.

   (a) For a given δ, solve the conditional (on z) objective that adjusts for potential misspecification. To evaluate this objective you may use the following formula: I

$$E\left[\exp\left(\mathsf{a} + \mathsf{b}W + \frac{\mathsf{c}}{2}W^2\right)\right] = \frac{1}{\sqrt{1-\mathsf{c}}}\exp\left[\mathsf{a} + \frac{\mathsf{b}^2}{2(1-\mathsf{c})}\right]$$

   where $\mathsf{c} < 1$ provided that W is a standard normally distributed random variable, You **do not** have to derive this formula, although it can be shown with a "complete-the-square" argument.

   (b) Find the corresponding minimizing conditional probability density over $\mathcal{W}$ conditioned on z for a given decision rule δ.

   (c) Compute the robustly optimal δ and the associated worst-case density. Does the decision rule depend on ξ? Does the worst-case density depend on δ? This outcome is very special driven by the underlying assumption of normality.