Homework for Econ 312 Chapter 2

May 5, 2026

1. Consider a three state Markov chain with transition matrix:

$$\mathbb{P} = \begin{bmatrix} .3 & .4 & .3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

   (a) Find the stationary distribution.

   (b) Is the process ergodic when initialized at this distribution? Explain.

2. Consider a first-order, n-dimensional, vector autoregression:

$$X_{t+1} = \mathbb{A}X_t + \mathbb{B}W_{t+1}$$

   where $\{W_{t+1}, t = 0, 1, ...\}$ is a multivariate standard normally distributed random vector. Let u' denote a non-degenerate row vector of $\mathbb{A}$:

$$\mathsf{u}'\mathbb{A} = \eta\mathsf{u}'.$$

   Suppose that $\{X_t\}$ is stationary.

   (a) Compute $\mathbb{E}(\mathsf{u}'X_{t+1} \mid X_t = x)$ using the eigenvector restriction.

   (b) Consider two cases.

      i. Suppose that $\eta \neq 1$. What does item (a) imply for $\mathbb{E}X_t$ in a stationary distribution?

      ii. Suppose that $\eta = 1$. What does item (a) imply for $\mathbb{E}X_t$ in a stationary distribution?

   (c) The covariance matrix for X_t in a stationary distribution should satisfy:

$$\Sigma = \mathbb{A}\Sigma\mathbb{A}' + \mathbb{B}\mathbb{B}'$$

Pre multiply this equation by u' and post multiply this equation by u.

$$\mathsf{u}'\Sigma\mathsf{u} = \mathsf{u}'\mathbb{A}\Sigma\mathbb{A}'\mathsf{u} + \mathsf{u}'\mathbb{B}\mathbb{B}'\mathsf{u}$$

Suppose that $\eta = 1$. Produce an implied equation involving only $\mathsf{u}'\Sigma\mathsf{u}$ and $\mathsf{u}'\mathbb{B}\mathbb{B}'\mathsf{u}$. What does this simplified equation imply for:[1]

$$\mathsf{u}'\mathbb{B}\mathbb{B}'\mathsf{u}.$$

(d) Suppose that $\mid \eta \mid > 1$. Using item (a), what can you say about the limit:

$$\lim_{t\to\infty} \mathbb{E}(\mathsf{u}'X_t \mid X_0 = x)$$

for $x \neq 0$?

3. A researcher considers two possible n state Markov process models with $n \times n$ transition matrices: $\mathbb{P}_1$ and $\mathbb{P}_2$. She or he is unsure as to which one generates the data that are observed. Suppose that the

$$\mathbb{M}_j = (1-\lambda)\sum_{\tau=0}^{\infty} (\mathbb{P}_j)^{\tau}$$

has strictly positive entries for $j = 1, 2$. A thoughtful research assistant who has taken Econ 312 suggests representing the model by studying a $2n \times 2n$ dimensional Markov process model with composite transition matrix:

$$\mathbb{P}_c = \begin{bmatrix} \mathbb{P}_1 & 0 \\ 0 & \mathbb{P}_2 \end{bmatrix}$$

where there is an augmented state vector that includes the model.

(a) Let a realization of the state vector for the composite state be a $2n$ dimensional coordinate vector where any vector with a one in the first n entries indicates that model one is generating the data and any vector with a one in the second n entries indicates that model two is generating the data. Explain why $\mathbb{P}_c$ can be used as the transition matrix with uncertainty as to which model is generating the data.

(b) Will

$$\mathbb{M}_c = (1-\lambda)\sum_{\tau=0}^{\infty} (\mathbb{P}_c)^{\tau}$$

[1] The result you show can actually be extended to the case where $\mid \eta \mid = 1$ where η could be complex. In this case, u may be a vector of complex numbers and the post multiplications should be by the complex conjugate of u. You do not have to demonstrate this in your answer.

have all entries that are strictly positive? Justify your answer.

(c) Let q_j be a nonnegative n-dimensional vector with entries that sum to one that satisfies:

$$(\mathsf{q}_j)' \, \mathbb{P}_j = (\mathsf{q}_j)'$$

for $j = 1, 2$. Interpret the q_j's. Use these two vectors to represent a family of solutions to

$$\mathsf{q}' \mathbb{P}_c = \mathsf{q}' \tag{1}$$

(d) Under which of the members of this family of solutions q_c to (1) will the Markov process with transition matrix $\mathbb{P}_c$ be ergodic when initialized at the solution q_c.

(e) The research assistant notes that the alternative solutions to (1) include the subjective prior for each model. Given such a solution, q_c, show how to back out the implied prior for each model.